Comparison of multiscale methods for the analysis of fine periodic electromagnetic structures

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Abstract — This paper deals with the application of multiscale finite element methods to the solution of electromagnetic-field problems including fine heterogeneous media. Two approaches are considered, respectively based on the introduction of modified shape functions and on the decomposition of the unknown field into a coarse and a fine term. Results show the capabilities of the considered multiscale methods of reproducing spatial behavior of field quantities in comparison with standard finite element solution.

I. INTRODUCTION

Fine periodic structures frequently appear in the analysis of electromagnetic problems. Typical examples are the heterogeneous magnetic and dielectric media (e.g. composite materials) whose components are suitably mixed to obtain desired macroscopic properties. Other examples of fine periodic structures are given by passive magnetic shields, where periodicity is determined by the presence of regular holes to enhance thermal energy dissipation.

The numerical analysis of electromagnetic problems including fine periodic structures is made difficult by the presence of several spatial scales, all affecting the overall response of the system. If standard numerical techniques are adopted to solve these multiscale problems, the number of unknowns of the resulting algebraic system becomes too large, rapidly reaching the limits of computing resources. For these reasons, multiscale techniques have been introduced with the aim of separating the different spatial scales, but keeping the influence of each scale on the other ones. Examples of application of the multiscale techniques can be found in different scientific and engineering disciplines, such as material science, mechanics, fluidodynamics and electromagnetics [1-4].

In this work, we have implemented and compared two multiscale finite element approaches, making reference to the Helmoltz equation with highly oscillatory coefficients, determined by material properties changing at the finer scale. The results obtained with the multiscale finite element methods will be also compared with those obtained by an alternative approach based on a homogenization technique with local correctors [5-7].

II. PROBLEM DESCRIPTION

We refer to a 2-D bounded domain S, with a coordinate system $s=(x_1, x_2)$. S is assumed to be highly periodic, that is complex conductivity $\tilde{\sigma}$ and magnetic permeability μ vary with period $Y =]0,Y_1[\times]0,Y_2[$. The spatial period Y is defined as the elementary cell, and constitutes an intermediate spatial scale between the smallest one (defined by the spatial variation of the electromagnetic properties within the cell) and the largest one (defined by the entire bounded domain S). Assuming the magnetic field directed along the perpendicular direction (z-axis) and considering sinusoidal supply conditions (angular frequency ω), the following weak form equation in the harmonic domain holds

$$\int_S \frac{1}{\tilde{\sigma}} \vec{\nabla} H \cdot \vec{\nabla} v \, ds + i\omega \int_S \mu H v \, ds = 0 \quad (1)$$

being $\vec{H} = H(x_1, x_2)\vec{u}_z$ the magnetic field (expressed as phasor). Non homogeneous Dirichlet boundary conditions ($H = H_0$ on ∂S) complete the problem description. Equation (1) is handled by multiscale finite element techniques, either by defining oscillating local shape functions at the elementary cell level (Method #1) or by obtaining the global solution as the sum of a fine and a coarse term (Method #2). In both methods a coarse mesh ($\Im_c$) is identified by the multiple repetition of the elementary cell (M_c coarse elements with N_c nodes) and a fine mesh ($\Im_f$) is defined within each cell (M_f coarse elements with N_f nodes for each cell). The standard computation of (1) would require the solution of a system of order $N_f \cdot N_c$.

A. Method #1

Considering a coarse element K, the following set of problems (1) is solved, one for each vertex i:

$$L\phi_i = 0 \quad \text{with} \quad \phi_i = \phi_i^{(0)} \text{ on } \partial K \quad (2)$$

being L the differential operator defined by problem (1) and ϕ_i the local shape function associated to vertex i of the coarse element K [1, 2]. Having determined the values of shape functions ϕ_i on each node of the fine mesh, the following coarse scale problem is solved:

$$\int_S \frac{1}{\tilde{\sigma}} \alpha \vec{\nabla} \phi \cdot \vec{\nabla} w \, ds + i\omega \int_S \mu \alpha \phi w \, ds = 0 \text{ with } \alpha = H_0 \text{ on } \partial S \quad (3)$$

being α the unknowns on the N_c nodes of the coarse mesh, and w the corresponding test functions. The value of H in a generic point of domain S is then obtained as $H = \sum_i \alpha_i \phi_i$, where rapid spatial oscillations are resolved by local shape functions ϕ_i. From the computational viewpoint, this method requires the preliminary solution of V systems (being V the number of coarse element vertices) of order N_f, plus the solution of a system of order N_c.

B. Method #2

The magnetic field H is written as the sum of a fine term H_f and a coarse term H_c [3, 4]. By defining coarse (w) and fine (v) test functions, two problems derives from (1):

$$\int_K \frac{1}{\tilde{\sigma}} \vec{\nabla} H_f \cdot \vec{\nabla} v \, ds + i\omega \int_K \mu H_f v \, ds =$$

$$= -\int_K \frac{1}{\tilde{\sigma}} \vec{\nabla} H_c \cdot \vec{\nabla} v \, ds - i\omega \int_K \mu H_c v \, ds \quad \text{with } H_f = 0 \text{ on } \partial K$$

$$\int_S \frac{1}{\tilde{\sigma}} \vec{\nabla} H_c \cdot \vec{\nabla} w \, ds + i\omega \int_S \mu H_c w \, ds +$$

$$\int_S \frac{1}{\tilde{\sigma}} \vec{\nabla} H_f \cdot \vec{\nabla} w \, ds + i\omega \int_S \mu H_f w \, ds = 0 \quad \text{with } H_c = H_0 \text{ on } \partial S \qquad (4)$$

In the first one, defined on the coarse element K, H_f is determined as a function of H_c. This relation is used in the second problem defined on the entire domain S. Since H_f is assumed to be null on boundary ∂K, H_c verifies the boundary conditions $H = H_0$ on ∂S. The solution of problems (4) provides H_c on each node of the coarse mesh and H_f on each node of the fine mesh.

The computational burden is similar to Method #1, requiring the preliminary solution of V systems (being V the number of coarse element vertices) of order N_f, plus the solution of a system of order N_c.

III. EXAMPLES

The two multiscale finite element methods have been compared in the analysis of two fine periodic structures. The first one simulates the cross section of a magnetic core made of soft ferrite, modeled by the multiple repetition of elementary cells, representing magnetic grains (size equal to 10 µm) surrounded by a dielectric layer (thickness equal to 50 nm). A sinusoidal magnetic field (amplitude H_0, frequency equal to 1 MHz) is applied perpendicularly to the cross section. The second example simulates a grid shield, where the square elementary cell (size equal to 50 mm) is constituted of conductive material and has an internal hole (size of 30 mm). A sinusoidal magnetic field (amplitude H_0, frequency equal to 50 Hz) is applied perpendicularly to the shield.

For both cases, the results obtained by the two multiscale finite element methods (MsFEM) are compared with the standard FEM solution and with the solution obtained by a homogenization technique with local correctors [7]. Figures 1 and 2 show the spatial distribution of magnetic field computed by the different approaches. The goodness of the spatial reconstruction is proved for both MsFEM methods.

In the full paper an extended numerical analysis will be presented, putting in evidence the potentiality of the considered multiscale methods.

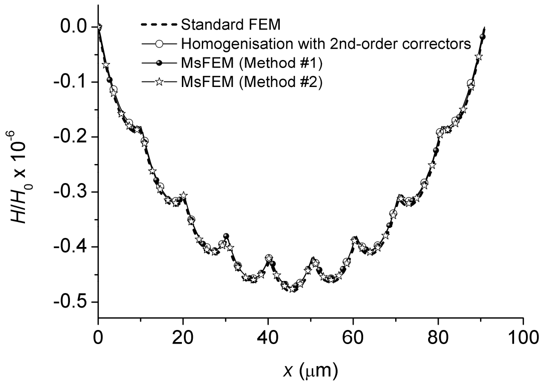

Fig. 1. Example #1: spatial distribution of the magnetic field along the material cross section.

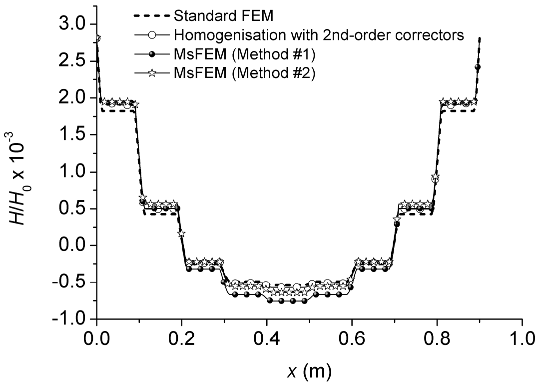

Fig. 2. Example #1: spatial distribution of the magnetic field along the planar grid shield.

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